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Quasi-stationary optical solitons with power law nonlinearity

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Abstract

The multiple-scale perturbation analysis is used to study the perturbed nonlinear Schrödinger equation, due to power law nonlinearity, that governs the propagation of solitons through an optical fibre. We have considered the perturbations due to the nonlinear damping and saturable amplifiers. A new definition of the phase of the soliton is introduced that captures the corrections to the pulse where the standard soliton perturbation theory fails. The numerical results support the analysis.

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1. Introduction

The dynamics of pulses propagating through optical fibres has been a major area of research given its potential applicability in all optical communication systems. It has been well established [3, 13, 19] that this dynamics is described, to first approximation, by the integrable nonlinear Schrödinger equation (NLSE). Here, the global characteristics of the pulse envelope can be fully determined by the method of inverse scattering transform (IST) and in many instances, the interest is restricted to the single pulse described by the one soliton form of the NLSE. Typically though, distortions of these pulses arise due to perturbations which are higher order corrections in the model as derived from the original Maxwell's equations [13] (examples are higher order linear and nonlinear dispersion terms, and two photon absorption), physical mechanisms not considered at first approximation (e.g. Raman effects) or external perturbations such as the lumped effect due to the addition of bandwidth-limited amplifiers in a communication line. Mathematically, these corrections are seen as perturbations of the NLSE and most of them have been studied thoroughly [7–9, 19] by regular asymptotic [14], soliton perturbation theory (SPT) [15] or Lie transform [19] methods. In particular, the last method has successfully addressed two important problems, where the two other methods fail: the effect of higher order dispersion in solitons and the long term dynamics of pulses in communication lines with periodic variation of the dispersion and nonlinear coefficient (the guiding centre model) [13]. In this paper, we will study the NLSE with power law nonlinearity as a generalization to the Kerr law nonlinearity. Various materials exhibit power law nonlinearities, including semiconductors [16, 17]. The dimensionless form of the NLSE with the power law of nonlinearity is given by

$$\mathrm{i}q_t + \tfrac{1}{2}q_{xx} + |q|^{2p}\, q = 0. \tag{1}$$

Here, we need to have $0 < p < 2$ to prevent wave collapse [18]. We also need to have the restriction $p \neq 2$ to avoid the self-focusing singularity issue [2, 6, 8, 14, 18]. We note that, for $p = 1$ in (1), we recover the NLSE with the Kerr law of nonlinearity. In (1), q represents the dimensionless optical field in the fibre core while t is the length of the fibre core and x represents the time. We note that (1) is not integrable by IST unless $p = 1$ in which case we recover the Kerr law nonlinearity. However, (1) supports solitons of the form [2, 8, 14]

$$q(x,t) = \frac{A}{\cosh^{\frac{1}{p}}[B(x - vt - \bar{x})]}\,\mathrm{e}^{(-\mathrm{i}\kappa x + \mathrm{i}\omega t + \mathrm{i}\sigma_0)} \tag{2}$$

where

$$\kappa = -v \tag{3}$$

and

$$\omega = \frac{B^2 - p^2\kappa^2}{2p^2} \tag{4}$$

with

$$B = A^p \left(\frac{2p^2}{1+p}\right)^{\frac{1}{2}}. \tag{5}$$

Here A is the amplitude of the soliton, B is the width of the soliton, v is its velocity, κ is the soliton frequency and ω is the wave number while $\bar{x}$ and σ_0 are the centre of the soliton and the centre of the soliton phase respectively.

The perturbed NLSE that will be considered in this paper is

$$\mathrm{i}q_t + \frac{1}{2}q_{xx} + |q|^{2p}\, q = \mathrm{i}\epsilon\left(\sigma q \int_{-\infty}^{x} |q|^2\,\mathrm{d}s - \delta|q|^{2m}\right). \tag{6}$$

In optics, ϵ is called the relative width of the spectrum that arises due to quasimonochromaticity [13] and $0 < \epsilon \ll 1$. Moreover, δ is the nonlinear damping or amplification coefficient [1, 7–9] depending on the sign and m could be 0, 1, 2. For $m = 0$, δ is the linear amplification or attenuation depending on whether δ is positive or negative. For $m = 1$, δ represents the two-photon absorption (or a nonlinear gain if $\delta > 0$). If $m = 2$, δ gives a higher order correction (saturation or loss) to the nonlinear amplification–absorption. Finally, σ is the coefficient of saturable amplifiers [1, 13, 14]. A model with saturation term included is more satisfactory from a physical point of view since stable soliton propagation is then ensured, in principle, over an indefinite propagation distance. In (6), if we set $\epsilon = 0$, we recover (1), the NLSE.

We shall study the perturbed NLSE, given by (6), by the method of quasi-stationarity in the next section. The motivation for this method of study is that, using this method, one can recover the results that are obtained by the SPT, which will we see. Moreover, the first order correction to the soliton can also be obtained using this method.

2. Quasi-stationary solution

The idea for solving the nonlinear evolution equations by the quasi-stationary method was first introduced in 1981 by Kodama and Ablowitz [2, 14]. Later, it was utilized to study the NLSE with Kerr law nonlinearity for Hamiltonian and non-Hamiltonian type perturbation [7–9]. This paper is an extension of quasi-stationarity to the case of NLSE with power law nonlinearity.

In this method of quasi-stationarity we study the perturbed NLSE of the type given by (6). We write the solution of the unperturbed NLSE given by (1) in terms of certain natural fast and slow variables with some external parameters that depend on the slow variables. It is necessary that we develop equations for these parameters by using appropriate conditions such as secularity conditions, also known as the Fredholm alternative (FA) [2, 14]. These secularity conditions allow us to compute the solution at the $O(\epsilon)$ level that satisfies suitable boundary conditions. However, as is standard in perturbation problems, there is still freedom in the solution. This is due to the fact that some terms in the solution at the $O(\epsilon)$ level can be absorbed at the leading order by shifting other parameters. The solution at the $O(\epsilon)$ level can be made unique by imposing additional conditions that reflect specific initial conditions or other normalizations. Finally, continuation to higher order is straightforward.

The main aim of this work is to implement a quasi-stationarity to (6) by assuming a solution of the form [7–9]

$$q = \hat{q}(\theta, X, T; \epsilon)\,\mathrm{e}^{\frac{\mathrm{i}}{\epsilon}\rho(X,T;\epsilon)} \tag{7}$$

where

$$\frac{\partial\theta}{\partial x} = 1 \qquad \frac{\partial\theta}{\partial t} = 0$$

and

$$X = \epsilon x \qquad T = \epsilon t.$$

Here, as defined, θ is a fast variable while X and T are the slow variables in space and time respectively. When we turn on the perturbation term of the NLSE, we have that the soliton parameters A, B and κ are slowly varying functions of space and time, namely $A = A(X, T)$, $B = B(X, T)$ and $\kappa = \kappa(X, T)$. The dependence of the unperturbed soliton phase on these parameters clearly suggests the slowly varying dependence as is given by (7).

We now substitute (7) in (6) and expand

$$\hat{q} = \hat{q}^{(0)} + \epsilon\hat{q}^{(1)} + \epsilon^2\hat{q}^{(2)} + \cdots$$

$$\rho = \rho^{(0)} + \epsilon\rho^{(1)} + \epsilon^2\rho^{(2)} + \cdots$$

$$v = v^{(0)} + \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \cdots$$

to get at the leading order

$$-\left\{\rho_T^{(0)} + \frac{1}{2}\big(\rho_X^{(0)}\big)^2\right\}\hat{q}^{(0)} + \frac{1}{2}\frac{\partial^2\hat{q}^{(0)}}{\partial\theta^2} + (\hat{q}^{(0)})^{2p+1} = 0 \tag{8}$$

and

$$\big(\rho_X^{(0)} - v^{(0)}\big)\frac{\partial\hat{q}^{(0)}}{\partial\theta} = 0. \tag{9}$$

Now, (9) implies

$$\rho_X^{(0)} = v^{(0)}. \tag{10}$$

We set

$$\frac{B^2}{2p^2} = \rho_T^{(0)} + \frac{1}{2}\big(\rho_X^{(0)}\big)^2 = \rho_T^{(0)} + \frac{1}{2}(v^{(0)})^2 \tag{11}$$

so that (8) changes to

$$-\frac{B^2}{2p^2}\hat{q}^{(0)} + \frac{1}{2}\frac{\partial^2 \hat{q}^{(0)}}{\partial\theta^2} + (\hat{q}^{(0)})^{2p+1} = 0 \tag{12}$$

whose solution is

$$\hat{q}^{(0)} = \frac{A}{\cosh^{\frac{1}{p}}[B(\theta - \bar{\theta})]} \tag{13}$$

where

$$B = A^p\left(\frac{2p^2}{1+p}\right)^{\frac{1}{2}} \tag{14}$$

and

$$\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}t} = v. \tag{15}$$

Thus, from (10) and (11) we see that the soliton frequency and the wave number at the first order agree with (4) and (5). At $O(\epsilon)$, we decompose $\hat{q}^{(1)} = \hat{\phi}^{(1)} + \mathrm{i}\hat{\psi}^{(1)}$ into its real and imaginary parts. Now the equations for $\hat{\phi}^{(1)}$ and $\hat{\psi}^{(1)}$ are respectively

$$-\frac{B^2}{2p^2}\hat{\phi}^{(1)} + \frac{1}{2}\frac{\partial^2 \hat{\phi}^{(1)}}{\partial\theta^2} + (2p+1)(\hat{q}^{(0)})^{2p}\hat{\phi}^{(1)} = \left(\rho_T^{(1)} + v^{(0)}\rho_X^{(1)}\right)\hat{q}^{(0)} - \frac{\partial^2 \hat{q}^{(0)}}{\partial\theta\partial X} \tag{16}$$

and

$$\begin{aligned}-\frac{B^2}{2p^2}\hat{\psi}^{(1)} + \frac{1}{2}\frac{\partial^2 \hat{\psi}^{(1)}}{\partial\theta^2} + (\hat{q}^{(0)})^{2p}\hat{\psi}^{(1)} &= -\frac{\partial\hat{q}^{(0)}}{\partial T} - v^{(0)}\frac{\partial\hat{q}^{(0)}}{\partial X} + \left(v^{(1)} - \rho_X^{(1)} - \sigma\right)\frac{\partial\hat{q}^{(0)}}{\partial\theta}\\ &\quad - \rho_{XX}^{(0)}\hat{q}^{(0)} - \delta(\hat{q}^{(0)})^{2m+1} + \sigma\hat{q}^{(0)}\int_{-\infty}^{x}(\hat{q}^{(0)})^2\,\mathrm{d}s.\end{aligned} \tag{17}$$

In an ideal soliton-based communication system, input pulses launched into the fibre should be unchirped in order to avoid shedding part of the pulse energy as a dispersive tail during the process of soliton formation [3]. So, in (17) we set $\rho_{XX}^{(0)} = 0$ to eliminate frequency chirp and thus obtain

$$\begin{aligned}-\frac{B^2}{2p^2}\hat{\psi}^{(1)} + \frac{1}{2}\frac{\partial^2 \hat{\psi}^{(1)}}{\partial\theta^2} + (\hat{q}^{(0)})^{2p}\hat{\psi}^{(1)} &= -\frac{\partial\hat{q}^{(0)}}{\partial T} - v^{(0)}\frac{\partial\hat{q}^{(0)}}{\partial X} + \left(v^{(1)} - \rho_X^{(1)} - \sigma\right)\frac{\partial\hat{q}^{(0)}}{\partial\theta}\\ &\quad - \delta(\hat{q}^{(0)})^{2m+1} + \sigma\hat{q}^{(0)}\int_{-\infty}^{x}(\hat{q}^{(0)})^2\,\mathrm{d}s.\end{aligned} \tag{18}$$

The FA [2, 7–9], applied to (16), gives

$$\frac{\partial B}{\partial X} = 0 \tag{19}$$

and

$$\rho_T^{(1)} + v^{(0)}\rho_X^{(1)} = 0 \tag{20}$$

whereas, if applied to (18), it gives

$$\begin{aligned}\frac{\mathrm{d}B}{\mathrm{d}T} &= -\frac{2\delta p}{2-p}\left(\frac{1+p}{2p^2}\right)^{\frac{m}{p}}\frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{p}\right)}\frac{\Gamma\left(\frac{m+1}{p}\right)}{\Gamma\left(\frac{m+1}{p}+\frac{1}{2}\right)}B^{\left(\frac{2m+p}{p}\right)}\\ &\quad + \frac{\sigma p}{2-p}\left(\frac{1+p}{2p^2}\right)^{\frac{1}{p}}\frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{p}\right)\Gamma\left(\frac{1}{2}\right)}B^{\frac{2}{p}}\int_{-\infty}^{\infty}\frac{1}{\cosh^{\frac{2}{p}}\tau}\left(\int_{-\infty}^{\tau}\frac{1}{\cosh^{\frac{2}{p}}s}\,\mathrm{d}s\right)\mathrm{d}\tau\end{aligned} \tag{21}$$

and

$$\rho_X^{(1)} = v^{(1)} - \sigma. \tag{22}$$

We see from (19) that B is a function of T alone and so also is A due to (5). Thus, by virtue of (5), we also get

$$\begin{aligned}\frac{\mathrm{d}A}{\mathrm{d}T} = &-\frac{2\delta}{2-p}A^{2m+1}\left(\frac{1+p}{2p^2}\right)^{\frac{1}{2p}}\frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{p}\right)}\frac{\Gamma\left(\frac{m+1}{p}\right)}{\Gamma\left(\frac{m+1}{p}+\frac{1}{2}\right)}\\ &+\frac{\sigma}{2-p}A^{3-p}\left(\frac{1+p}{2p^2}\right)^{\frac{2p}{p+1}}\frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{p}\right)\Gamma\left(\frac{1}{2}\right)}\int_{-\infty}^{\infty}\frac{1}{\cosh^{\frac{2}{p}}\tau}\left(\int_{-\infty}^{\tau}\frac{1}{\cosh^{\frac{2}{p}}s}\,\mathrm{d}s\right)\mathrm{d}\tau.\end{aligned} \tag{23}$$

We note that equations (21) and (23) represent the adiabatic parameter dynamics of the soliton width and amplitude respectively in the presence of the perturbation terms. We note that these can also be recovered by the SPT [8]. However, the relations (19)–(22) cannot be obtained by the SPT. This is where the SPT fails. Now (16), by virtue of (19) and (20), reduces to

$$-\frac{B^2}{2p^2}\hat{\phi}^{(1)} + \frac{1}{2}\frac{\partial^2\hat{\phi}^{(1)}}{\partial\theta^2} + (2p+1)(\hat{q}^{(0)})^{2p}\hat{\phi}^{(1)} = 0 \tag{24}$$

while (18), by virtue of (19) and (22), gives

$$-\frac{B^2}{2p^2}\hat{\psi}^{(1)} + \frac{1}{2}\frac{\partial^2\hat{\psi}^{(1)}}{\partial\theta^2} + (\hat{q}^{(0)})^{2p}\hat{\psi}^{(1)} = -\frac{\partial\hat{q}^{(0)}}{\partial T} - \delta(\hat{q}^{(0)})^{2m+1} + \sigma\hat{q}^{(0)}\int_{-\infty}^{x}(\hat{q}^{(0)})^2\,\mathrm{d}s. \tag{25}$$

Finally the solutions of (24) and (25) are respectively

$$\hat{\phi}^{(1)} = 0 \tag{26}$$

and

$$\begin{aligned}\hat{\psi}^{(1)} = &-\frac{2B^{\frac{1}{p}}}{p}\left(\frac{1+p}{2p^2}\right)^{\frac{1}{2p}}\left[\frac{\partial\bar{\theta}}{\partial T}\frac{\phi}{\cosh^{\frac{1}{p}}\phi}\int^{\phi}\cosh^{\frac{2}{p}}s_2\left(\int^{s_2}\frac{\tanh s_1}{\cosh^{\frac{2}{p}}s_1}\,\mathrm{d}s_1\right)\mathrm{d}s_2\right.\\ &-\frac{1}{B^2}\frac{\mathrm{d}B}{\mathrm{d}T}\int^{\phi}\cosh^{\frac{2}{p}}s_2\left(\int^{s_2}\frac{\tanh s_1}{\cosh^{\frac{2}{p}}s_1}\,\mathrm{d}s_1\right)\mathrm{d}s_2\\ &\left.+\frac{1}{B^3}\frac{\mathrm{d}B}{\mathrm{d}T}\int^{\phi}\cosh^{\frac{2}{p}}s_2\left(\int^{s_2}\frac{\tanh s_1}{\cosh^{\frac{2}{p}}s_1}\,\mathrm{d}s_1\right)\mathrm{d}s_2\right]\\ &-2\delta B^{\frac{2m-2p+1}{p}}\left(\frac{1+p}{2p^2}\right)^{\frac{2m+1}{2p}}\int^{\phi}\cosh^{\frac{2}{p}}s_2\left(\int^{s_2}\frac{\tanh s_1}{\cosh^{\left(\frac{2m+2}{p}\right)}s_1}\,\mathrm{d}s_1\right)\mathrm{d}s_2\\ &+2\sigma\frac{A^3}{B^3}\frac{1}{\cosh^{\frac{1}{p}}\phi}\int^{\phi}\cosh^{\frac{2}{p}}s_3\left(\int^{s_3}\frac{1}{\cosh^{\frac{2}{p}}s_2}\left(\int_{-\infty}^{s_2}\frac{1}{\cosh^{\frac{2}{p}}s_1}\,\mathrm{d}s_1\right)\mathrm{d}s_2\right)\mathrm{d}s_3\end{aligned} \tag{27}$$

where we have used $\phi = B(\theta - \bar{\theta})$. The $O(\epsilon)$ solution of (6) finally is

$$q \approx P\,\mathrm{e}^{\mathrm{i}\psi} \tag{28}$$

where

$$P = \hat{q}^{(0)}$$

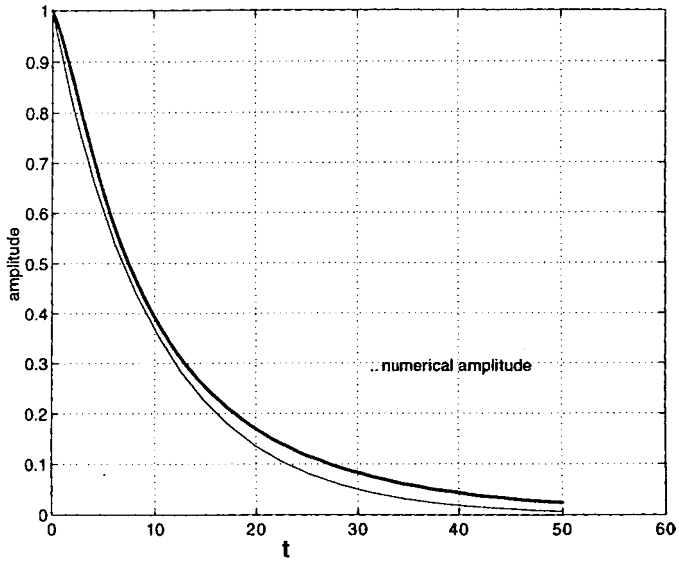

Figure 1. Amplitude variation for $p = 1$, $m = 0$, $\delta = 0.5$.

and

$$\psi = \epsilon Q(\theta) + \frac{1}{\epsilon}\rho(X, T)$$

with

$$Q(\theta) = \hat{\psi}^{(1)}/\hat{q}^{(0)}.$$

Equation (28) is our required quasi-stationary optical soliton of the perturbed NLSE with the power law of nonlinearity.

We note that, by virtue of (21) and (23), the quasi-stationary soliton given by (28) travels through the optical fibre with group velocity [13] at a fixed amplitude ($\bar{A}$) and width ($\bar{B}$) that are given by

$$\bar{A} = \left[\frac{\sigma I_p}{2\delta}\left(\frac{1+p}{2p^2}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{m+1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{m+1}{p}\right)\Gamma\left(\frac{1}{2}\right)}\right]^{\frac{1}{2m+p-2}} \tag{29}$$

and

$$\bar{B} = \left[\frac{\sigma I_p}{2\delta}\left(\frac{2p^2}{1+p}\right)^{\frac{m-1}{p}} \frac{\Gamma\left(\frac{m+1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{m+1}{p}\right)\Gamma\left(\frac{1}{2}\right)}\right]^{\frac{p}{2m+p-2}} \tag{30}$$

with

$$I_p = \int_{-\infty}^{\infty} \frac{1}{\cosh^{\frac{2}{p}}\tau}\left(\int_{-\infty}^{\tau} \frac{1}{\cosh^{\frac{2}{p}}s}\,\mathrm{d}s\right)\mathrm{d}\tau.$$

So, there exists a steady state soliton, where the amplitude and the frequency (velocity) get locked at fixed values in a medium where the energy growth rate and the energy losses are fully compensated against each other [7]. One can see this new physical phenomenon, for the first time, in this paper for optical solitons with power law nonlinearity. This model can be very effective in the applied soliton community for studying the propagation of solitons

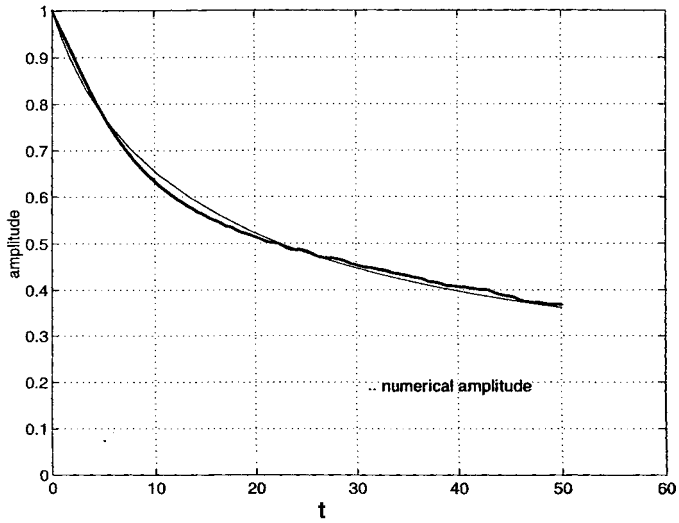

Figure 2. Amplitude variation for $p = 1$, $m = 1$, $\delta = 0.5$.

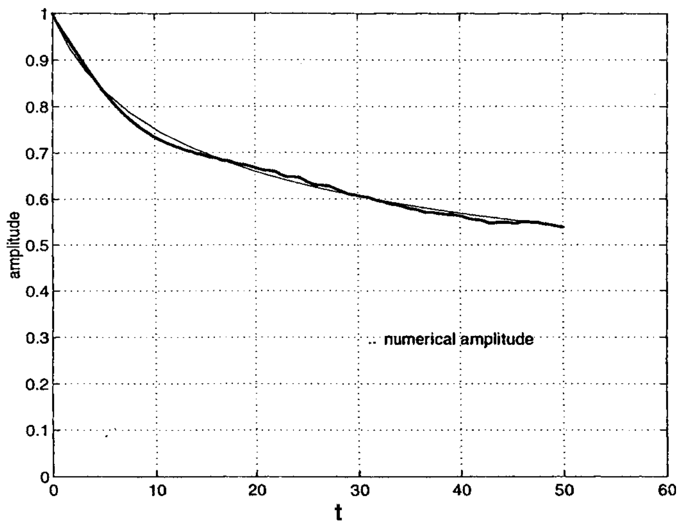

Figure 3. Amplitude variation for $p = 1$, $m = 2$, $\delta = 0.5$.

through an optical fibre in power law media. It will serve as the most effective means of a bit carrier in optical soliton communications.

3. Numerical simulation

We have now carried out the direct numerical simulation of equation (11). Here we have used a hyperbolic secant profile for the soliton. The fast Fourier transform (FFT) of the profile in the space variable is used. The different modes of the FFT are studied. The program proceeds in the time step by Picard iteration. The evolution of the nonlinear terms is carried out by the convolution integrals. The iteration ceases when the difference of values between successive iterations is at $O(h^2)$ where h is the time step.

In the following figures, we have obtained the numerical and the analytical variation of the amplitude of the perturbed soliton. They are plotted on the same set of axes for a direct comparison.

Figure 1 shows the numerical and analytical variations of the soliton amplitude given by (23). Here, the special case $p = 1, m = 0$ is plotted for $\delta = 0.5$.

Now, figure 2 gives numerical and analytical variations of the soliton amplitude in (23) for $p = 1, m = 1, \delta = 0.5$.

Finally, in figure 3 we have the numerical and analytical variations of the amplitude of the soliton for $p = 1$ with $m = 2$ and $\delta = 0.5$.

Thus, in figures 1–3 we see that the agreement between the theory and the numerics of the variation of the amplitude of the soliton in the three cases is very good.

4. Conclusions

In this paper we have obtained a solution of the perturbed NLSE up to $O(\epsilon)$ by the method of multiple scale perturbation expansion. Our solution matches the ones that were obtained in previous works for the special cases of this problem [2, 11, 12]. It also captures the variation of the soliton parameters, up to $O(\epsilon)$, due to the perturbation term that cannot be recovered by the soliton perturbation theory. Also, the numerical results support the analytical argument.

Although there have been studies on the solitons in other types of non-Kerr law media such as the parabolic law of nonlinearity [4, 20], studies due to dual power law, threshold nonlinearity, saturable media and others are still awaited. Moreover, the radiation due to these perturbation terms with power law nonlinearity is yet to be studied.

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